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Numerical Validation of Time-Efficient ECAP Simulations Based on Energy Criteria

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Abstract: Equal Channel Angular Pressing (ECAP) is a widely used severe plastic deformation technique for producing ultrafine-grained metals. Numerical modelling plays an essential role in analyzing the process mechanics. The purpose of this study is to reduce the computational cost of explicit ECAP simulations by applying mass scaling while ensuring that the accuracy of the results is not compromised. The methodology includes explicit finite-element modelling with controlled mass scaling and verification by monitoring internal, kinetic, and total energies throughout deformation. The ratio of kinetic to internal energy is used as the primary indicator of acceptable quasi-static behavior. The results show that the applied mass-scaling level significantly reduces computational time without altering the predicted stress distribution, plastic strain fields, or forming forces, provided that the kinetic energy remains within recommended limits. The application of mass scaling reduced simulation time by more than two orders of magnitude while preserving quasi-static conditions, as indicated by a consistently low kinetic-to-internal energy ratio. The study concludes that energy-based verification offers a reliable framework for applying mass scaling in ECAP simulations while preserving the physical validity of the results.

Keywords: ECAP; mass scaling; severe plastic deformation; quasi-static behavior

1. Introduction

Metallic materials with fine- or ultrafine-grained (UFG) microstructures constitute a distinct class of engineering materials characterized by superior mechanical properties, including increased strength, improved toughness, and enhanced resistance to fatigue and fracture. These benefits arise primarily from grain refinement, a phenomenon theoretically described by the Hall–Petch relationship, which states that the yield strength of a material increases as grain size decreases. Therefore, techniques capable of producing such refined microstructures have gained substantial scientific and industrial interest [1-5].

Among the most effective approaches for achieving UFG structures are Severe Plastic Deformation (SPD) methods, which introduce extensive plastic strains while maintaining the specimen's external geometry. Within the SPD family, Equal Channel Angular Pressing (ECAP) has emerged as one of the most widely studied and practically applied techniques [6-7]. In ECAP, a metallic billet is forced through two channels of identical cross-section that intersect at a prescribed angle, typically between 90° and 120° [8-9]. As the billet passes through the

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intersection, it undergoes intense shear deformation that drives significant plastic flow and progressive grain refinement. Because the billet's cross-section remains unchanged, the process can be repeated multiple times to accumulate strain and improve microstructural homogeneity [10-11].

Aluminum and its alloys are particularly suitable for ECAP due to their high plasticity, low density, and broad technological relevance. ECAP processing enables substantial improvements in the strength and hardness of aluminum while maintaining acceptable ductility, making it an attractive method for producing lightweight, high-performance components [12-15]. However, the ECAP process involves complex deformation mechanisms, including nonlinear material behavior, frictional effects, and intricate contact interactions, which pose significant challenges for experimental investigation [16].

For these reasons, computational simulations, especially those based on the Finite Element Method (FEM), have become indispensable tools for analyzing ECAP. Software platforms such as Abaqus enable detailed examination of plastic strain distribution, stress evolution, and the influence of key process parameters, providing insights that are often difficult to obtain experimentally [16-17]. Despite the widespread use of numerical modelling, a review of the literature indicates that the optimization of ECAP simulation procedures has received insufficient attention. Existing studies rarely address the challenges posed by long computation times or propose strategies to reduce them.

While numerous numerical studies have investigated ECAP, with an emphasis on strain distribution, stress evolution, and microstructural refinement, significantly less attention has been devoted to optimizing computational efficiency and developing numerical verification strategies. In particular, the application of mass scaling in explicit ECAP simulations is often mentioned but rarely examined systematically. The present work addresses this gap by analyzing the influence of mass scaling on simulation time and result accuracy, and by combining an energy-based verification approach to establish acceptable scaling limits. This methodology provides practical guidance for reducing computational cost while maintaining physically consistent simulation results.

Simulation optimization is vital for several reasons: shorter computation times enable faster iteration and process refinement, allowing more variants to be tested within a shorter period; additionally, reducing computational demand lowers costs and accelerates engineering decision-making. Therefore, the objective of the present study is to reduce ECAP simulation time while keeping deviations in the results within acceptable limits, ultimately contributing to a more efficient and reliable computational workflow for modelling the ECAP process.

2. Materials and Methods

In this study, a numerical simulation of the Equal Channel Angular Pressing (ECAP) process was performed in Abaqus 2023 using an explicit analysis method. A consistent SI-based unit system was adopted, with length expressed in millimeters (mm), force in newtons (N), mass in tons (10^3 kg), time in seconds (s), stress in megapascals (MPa = N/mm^2), energy in millijoules (mJ), and density in tons per cubic millimeter (ton/mm^3).

2.1. Rigid parts

To reduce overall computational complexity while retaining an accurate representation of contact interactions, the numerical model was limited to the internal channel surface of the die, which constitutes the functional interface governing the billet's plastic flow. The channel, as seen in Figure 1a, configured with an intersection angle of 120°, was idealized as a rigid body and modelled through a shell-based geometric abstraction. Discretization was performed using R3D4 rigid shell elements, four-node entities designed without integration points or internal degrees of freedom, thereby significantly reducing computational demand while ensuring that the die's kinematic constraints and contact behaviour were faithfully represented.

The punch, as shown in Figure 1b, was modeled as a rigid three-dimensional body, since its deformation is negligible relative to that of the workpiece. It was discretized using shell rigid elements, thereby providing a consistent and computationally efficient definition of the tool while accurately capturing the kinematic boundary conditions imposed on the billet.

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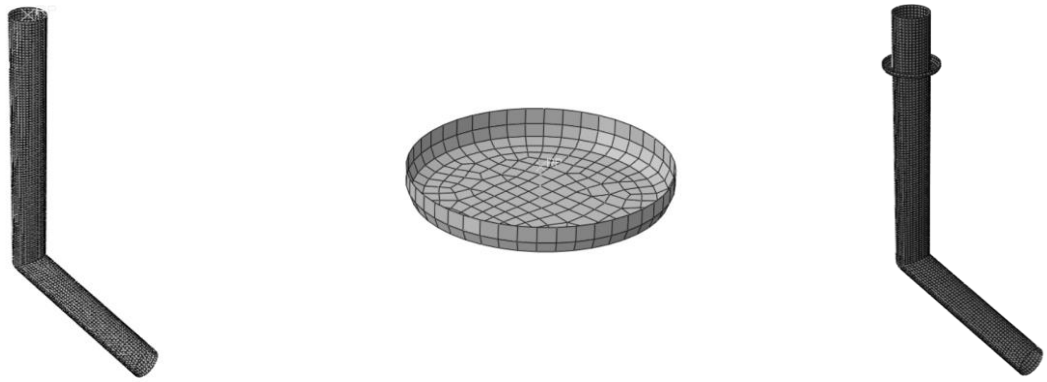

a) b) c)

Figure 1: a) Channel after discretisation, b) Punch after discretization, c) Punch and Channel assembly [18]

2.2. Deformable part

The workpiece, as shown in Figure 2a, was modeled as a deformable cylindrical body with dimensions of Ø10 mm × 80 mm and assigned the elastic properties of aluminum (Al99.5). The material's plastic behavior was characterized using an actual stress–strain curve, the values of which are provided in Table 1. Finite element discretization was performed using C3D8R elements, which are eight-node linear brick elements with reduced integration, providing an efficient balance between computational cost and accuracy for large-deformation problems.

a) b)

Figure 2: a) Workpiece after discretization, b) Workpiece, Punch, and channel assembly (section view) [18]

Table 1. Plastic Flow Curve for Al99.5 [19]

Plastic Flow Stress, σ /MPa	Logarithmic Strain, ε
55.16	0.00
103.43	0.20
124.11	0.40
137.90	0.60
151.69	0.80
179.27	1.00
193.06	1.20
206.85	1.40
234.43	1.60
262.01	1.80
275.80	2.00

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2.3. Interaction

In the ECAP simulations, two main contact interactions were defined: between the workpiece and the die, and between the punch and the workpiece. A surface-to-surface formulation was applied in both cases, with the rigid parts (die and punch) modelled as master surfaces and the deformable workpiece as the slave surface.

Contact behaviour was implemented using the penalty method, which efficiently enforces contact constraints despite small surface penetrations. This approach is well-suited to highly nonlinear problems with large deformations, as it provides good numerical stability and reduces computational cost. Friction within the channel was modelled using a Coulomb friction coefficient of 0.1, which represents realistic interface conditions between the workpiece and the die during severe plastic deformation.

2.4. Boundary conditions

In the ECAP simulations, the die was modelled as a stationary rigid body. To ensure this condition, an encastre boundary condition was applied to the reference point associated with the die geometry, thereby constraining all translational and rotational degrees of freedom and preventing any movement or rotation of the channel during the analysis. For the punch, all degrees of freedom at the reference point were fixed, except along the Y-axis, which represents the longitudinal direction of the channel. A prescribed displacement was applied in this direction to simulate the punch motion and drive the material through the channel.

2.5. Simulation Time and Output Data

In the Step module, the total simulation time was defined as 40 seconds, corresponding to the duration of a single ECAP pass. This choice enabled the process to be represented under realistic time-dependent conditions. Output requests were configured to provide detailed data for post-processing. Field outputs included essential variables such as stress, strain, and equivalent plastic strain (PEEQ). For the history output, the reaction force was monitored at the punch reference point, allowing assessment of forming loads over time. At the same time, the entire workpiece was used as the domain for tracking the main energy components: internal energy (ALLIE), kinetic energy (ALLKE), and artificial strain energy (ALLAE). In this way, both mechanical response and energy balance were captured, ensuring that the simulation results could be validated against physical criteria [20].

3. Results

The numerical simulation was conducted under 11 defined conditions. The initial condition was executed without mass scaling, while the remaining 10 conditions employed varying scale factors. The objective was to monitor the reduction in simulation time and evaluate the results, with particular focus on the force–time diagram for each simulation case. Specific attention was given to assessing the extent to which the use of mass scaling factors influenced computational efficiency, while potentially compromising the accuracy of the results and the physical fidelity of the observed behavior.

3.1. Reduction in computational time

Table 2 shows the influence of mass scaling on the total simulation time. Slight deviations from a strictly monotonic decrease in computation time are attributed to variations in contact interactions and adaptive time increment adjustments during the explicit analysis. The reference simulation without mass scaling required 4800 minutes. Introducing mass scaling resulted in a continuous reduction in computation time across all scaling factors tested. For lower mass-scaling factors, the simulation time was reduced to several hundred minutes; for higher factors, it decreased further. The shortest simulation time, 26 minutes, was achieved using a mass scaling factor of 100 000. Overall, mass scaling reduced the simulation time by more than two orders of magnitude compared to the reference case.

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Table 2. Mass scaling impact on simulation time

Mass scale factor	Simulation time / min
No scale factor	4800
150	560
300	480
500	314
1 000	348
5 000	247
10 000	131
25 000	94
50 000	65
75 000	55
100 000	26

3.2. Verification before mass scaling

Figure 3 shows the evolution of the punch force over time during ECAP. At the beginning of the simulation, the force remains close to zero, corresponding to the initial contact and positioning of the billet. A rapid increase in force follows as the billet enters the channel intersection, reaching a maximum of approximately 130 kN. After the peak, the force gradually decreases with time, indicating a transition from initial severe deformation to a more stable material flow through the die. Minor force fluctuations observed at this stage are attributable to frictional effects and ongoing plastic deformation. Overall, the shape of the force–time response is consistent with trends reported in the literature [21-22] and reflects the characteristic behaviour of the ECAP process.

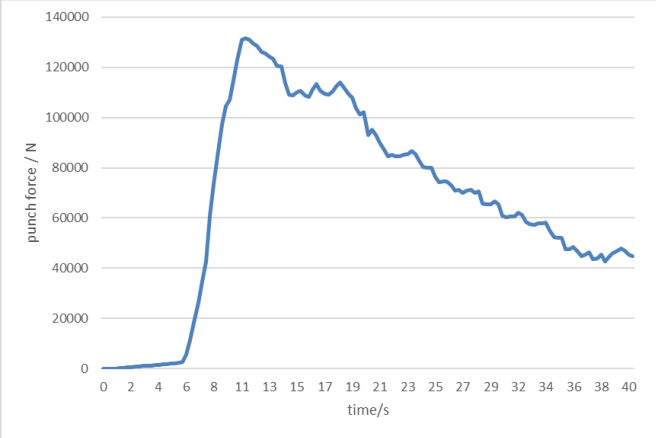

Figure 3. Force-time graph without mass scaling [18]

Figure 4 presents the evolution of the primary energy components during the simulation. The internal energy (ALLIE) increases continuously throughout the process, indicating that the majority of the externally applied work is dissipated through plastic deformation of the billet. In contrast, the kinetic energy (ALLKE) remains negligible throughout the simulation, whereas the total energy (ETOTAL) exhibits a stable, consistent trend.

Such an energy distribution confirms stable numerical behaviour and indicates that the process is governed by quasi-static conditions. To ensure an objective assessment of quasi-static behaviour, a quantitative criterion was introduced. The simulation was considered quasi-static when the ratio of kinetic energy to internal energy remained below 5% throughout the entire deformation process [23]. This threshold provides a clear and reproducible validation measure, ensuring that inertial effects do not significantly influence the numerical results.

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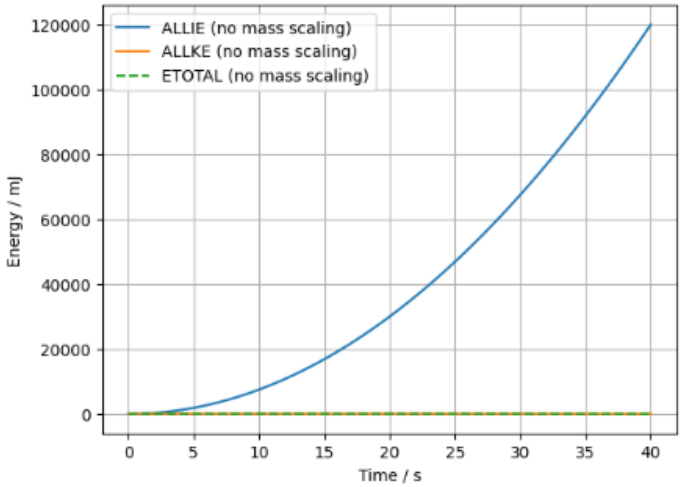

Figure 4. Energy analysis graph without mass scaling factor

3.3. Verification after utilisation of mass scaling

Figure 5 compares punch force–time responses for different mass-scaling factors, including the reference case without mass scaling. All curves exhibit a similar overall trend: a rapid increase in punch force as the billet enters the channel intersection, followed by a peak force of approximately 120–130 kN and a gradual decrease as material flow continues.

For mass scaling factors up to 100 000, the force–time curves remain closely aligned with the reference response, indicating that the process's general mechanical behaviour is preserved. Minor differences in peak force magnitude and curve smoothness can be observed at higher scaling factors; however, the overall force evolution remains comparable. These results demonstrate that mass scaling significantly reduces computational time while maintaining a consistent force response within acceptable limits.

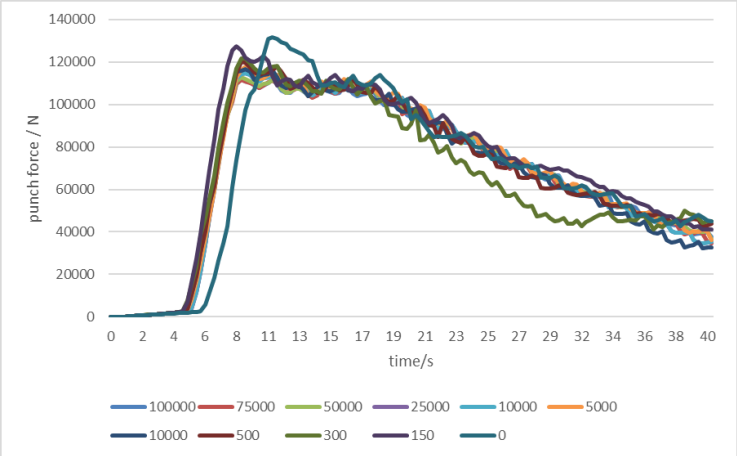

Figure 5. Punch force over time for simulations conducted with and without mass scaling [18]

Figure 6 compares the punch force–time response obtained without mass scaling (blue curve) with the average force–time response calculated from all simulations performed with mass scaling (red curve). Both curves show a similar overall trend, characterised by a rapid increase in punch force as the billet enters the channel intersection, followed by a peak force and a gradual decrease during continued deformation.

The mass-scaled response closely follows the reference curve without mass scaling, indicating that mass scaling does not significantly alter the global force evolution during ECAP. Minor differences in peak force

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magnitude and curve smoothness are observed, particularly during the initial loading phase; however, overall agreement between the curves indicates that the mechanical response remains consistent under mass scaling.

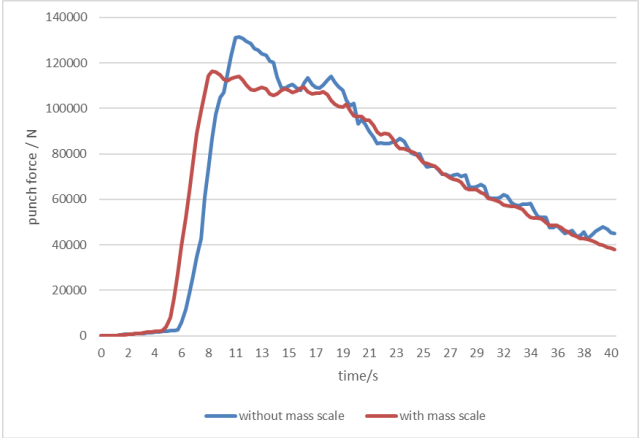

Figure 6. Comparison of the results without mass scale and the arithmetic mean of scaled results [18]

Figure 7 shows the evolution of internal energy (ALLIE), kinetic energy (ALLKE), and total energy (ETOTAL) for two representative cases: Case 1 without mass scaling and Case 2 with a mass scaling factor of 100 000. In both cases, internal energy increases continuously throughout the simulation and represents the dominant energy component. Slight differences in the slope and final magnitude of the internal energy are observed between the two cases, with Case 2 exhibiting a slightly lower energy level.

Kinetic energy remains very low throughout the simulation in both cases, whereas total energy follows a similar low-level trend. No abrupt changes or instabilities are observed in energy evolution. The qualitative behavior of all energy components is identical for both cases, indicating consistent numerical response across the analyzed configurations.

Energy evolution was evaluated for all applied mass-scaling factors; however, only representative cases are presented for clarity, as all simulations exhibited qualitatively similar energy behavior.

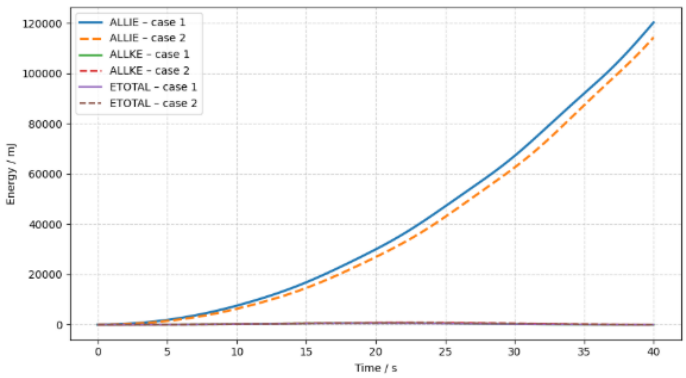

Figure 7. Comparison of the energy behaviour without the mass scale and the mass scale factor 100 000

4. Discussion

The results demonstrate that mass scaling can effectively accelerate explicit ECAP simulations while preserving the key characteristics of the process, provided that its application is carefully controlled. A significant reduction in computational time was observed with increasing mass-scaling factors, confirming the efficiency of this approach for explicit analyses involving fine meshes and complex contact conditions. However, the benefits of mass scaling are accompanied by essential limitations that must be acknowledged.

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The force–time responses obtained with mass scaling up to a factor of 100 000 show good agreement with the reference simulation performed without mass scaling. Although minor differences in peak force magnitude and curve smoothness are observed, particularly at higher scaling levels, the overall force evolution remains comparable. These deviations, although limited, indicate that mass scaling affects the numerical response and should therefore be applied within a validated range.

The observed similarity in the force–energy evolution supports time responses across different mass-scaling levels. The apparent dominance of internal energy over kinetic energy indicates that the deformation process remains governed by quasi-static conditions. This energy balance confirms that the applied mass scaling does not introduce significant inertial effects and that the predicted punch forces reliably reflect the underlying material response.

Energy-based analysis proved essential for identifying the limits of physically acceptable mass scaling. In all valid cases, internal energy dominated the system response, while kinetic and total energies remained negligible, confirming quasi-static conditions. Nevertheless, slight variations in the rate of internal energy accumulation were observed between the reference and mass-scaled simulations. These differences indicate that mass scaling alters the system's numerical inertia and may gradually affect the energy balance if excessive scaling is applied.

Based on the combined evaluation of force response and energy evolution, the present study identifies a mass scaling factor of 100 000 as a practical upper limit for the specific ECAP configuration considered here, including the material model, mesh resolution, and process parameters. Beyond this threshold, deviations from expected physical behavior become increasingly pronounced, reducing the reliability of the results. Therefore, mass scaling should not be treated as a purely numerical tool but rather as a modelling parameter that requires systematic validation. It should be emphasized that the main contribution of this work lies in demonstrating an energy-based procedure for validating mass scaling in ECAP simulations, rather than in defining a universal mass-scaling value.

Overall, the findings emphasize that while mass scaling offers substantial computational advantages, its application must be accompanied by rigorous energy-based verification. This approach ensures that gains in efficiency do not compromise the physical relevance of the simulation, thereby providing a reliable framework for the numerical analysis and optimization of ECAP processes.

5. Conclusions

Computational simulations of the ECAP process are often time-consuming due to complex material behavior, fine meshes, and nonlinear contact conditions. This study demonstrates that mass scaling in Abaqus/Explicit is a practical approach for reducing computational time without significantly affecting result accuracy, provided that appropriate limits are applied.

The validity of the numerical model was assessed using an energy-based approach. For all acceptable mass-scaling factors, the kinetic energy remained low relative to the internal energy, confirming quasi-static conditions. At the same time, the evolution of internal energy indicates that the applied work was predominantly dissipated through plastic deformation. These observations are consistent with results reported in the literature and support the reliability of the adopted modelling strategy.

It should be emphasized that the acceptable mass-scaling factor is not universal and depends on the specific process conditions. For harder materials, smaller ECAP channel angles, or higher pressing speeds, increased inertial effects are expected, necessitating stricter mass-scaling limits to preserve quasi-static behavior. Therefore, the admissible mass-scaling threshold must be determined individually for each configuration using energy-based validation.

Overall, the results confirm that mass scaling, when combined with systematic energy monitoring, can be safely used to improve computational efficiency in ECAP simulations. The presented approach provides a practical framework for reducing simulation time while maintaining physically consistent results and can be extended to other ECAP configurations, provided that the mass-scaling level is carefully validated for each case.

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